Meson-baryon bound states in a (2+1)-dimensional strongly coupled lattice QCD model

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We consider bound states of a meson and a baryon (meson and antibaryon) in lattice QCD in a Euclidean formulation. For simplicity, considering the + parity sector we analyze an SU(3) theory with a single flavor in 2+1 dimensions and two-dimensional Dirac matrices. We work in the strong coupling regime, i.e., in a region of parameters such that the hopping parameter κ is sufficiently small and $\kappa \gg g_0^{-2}$, where g_0^{-2} is the pure gauge coupling. There is a meson (baryon) particle with asymptotic mass $-2\ln\kappa$ ($-3\ln\kappa$) and an isolated dispersion curve. Here, in a ladder approximation, we show that there is no meson baryon (or meson-antibaryon) bound state solution to the Bethe-Salpeter equation up to the meson-baryon threshold ($\sim -5\ln\kappa$). The absence of such a bound state is an effect of a spatial range-one repulsive potential that is local in space at order κ^3, i.e., the leading order in the hopping parameter κ.

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In quantum chromodynamics (QCD), it is fundamental to establish on a rigorous basis the low energy-momentum (e-m) spectrum of particles and their bound states, and in particular, to show the existence of mesons and baryons, their bound states, and scattering. One way to study these properties is to use a lattice regularization of the continuum.

Nowadays, much attention has been paid to the so-called pentaquark state since its experimental discovery (see Ref. [1] and references therein). Here we search for these particles as bound states of a meson and a baryon in a simplified version of lattice QCD with only one flavor and at strong coupling.

Some numerical investigations were also reported on the existence of particles in lattice QCD [2–4], but these works do not provide us with a spectral representation for the correlation functions involved. This type of representation is crucial if we want to establish the correspondence between points in the energy-momentum spectrum and singularities of correlation functions as we do here.

Recently, we started the investigation of the particle spectrum in lattice QCD. The existence of baryons was shown in a 2+1 imaginary-time formulation of lattice SU(3) QCD, with 2×2 Pauli spin matrices and one quark flavor and for small plaquette coupling g_0^{-2} and small hopping parameter κ, such as $0<g_0^{-2}\ll\kappa\ll 1$ [5]. In Ref. [6] the existence of baryons was shown for (2+1)- and (3+1)-dimensional one-flavor lattice QCD, using 4×4 Dirac spin matrices. Similar results for mesons are reported in Ref. [7]. The baryon (meson) particle asymptotic mass is $-3\ln\kappa$ ($-2\ln\kappa$), where by asymptotic mass we mean $\lim_{\kappa\to 0} m_\kappa^b/(-3\ln\kappa)=1$ (m_κ^b being the baryon mass) and $\lim_{\kappa\to 0} m_\kappa^m/(-2\ln\kappa)=1$ (m_κ^m being the meson mass), which is associated with isolated dispersion curves in the e-m spectrum. The mass splitting for these particles is also obtained.

In Ref. [8], the question concerning the existence of baryon-baryon bound states in the e-m spectrum was analyzed for the (2+1)-dimensional case up to the two-baryon threshold ($\sim -6\ln\kappa$). The absence of such a bound state is an effect of a local repulsive potential at order κ^2. In Ref. [9], we considered the question of the existence of meson-meson bound states up to the two-meson threshold ($\sim -4\ln\kappa$). Through the effect of a nonlocal repulsive potential in space at order κ^2 we have found no bound state. These results were obtained using a lattice version of the Bethe-Salpeter (BS) equation. We also mention that in both cases the potential obtained is range-one and energy independent.

Here we consider the existence of meson-baryon bound states below the meson-baryon threshold ($\sim -5\ln\kappa$) in the + parity sector. Again, we employ the BS equation and use the spectral representations for the two-point meson [9] and baryon [8] functions, and introduce the four-point function for meson-baryon states. We find that the dominant interaction between a meson and a baryon occurs at order κ^3 and is a repulsive spatial range-one energy-independent local potential. At the end, we show the correspondence between the relative coordinate BS equation for zero system momentum and the one-particle lattice Schrödinger resolvent equation with a range-one local potential.

Here we work with the same lattice QCD model as in Ref. [5]. The partition function is given formally by $Z=\int e^{-S(\psi,\bar\psi,g)}d\psi d\bar\psi d\mu(g)$, where the action $S(\psi,\bar\psi,g)$ is

$$S(\psi,\bar\psi,g) = (\kappa/2)\sum \bar\psi_{\alpha,a}(u)\Gamma_{\alpha\beta}^{\epsilon e^\mu}(g_{u,u+\epsilon e^\mu})_{ab}\psi_{\beta,b}(u+\epsilon e^\mu) + \sum_{u\in\mathbb{Z}_0^3}\bar\psi_{\alpha,a}(u)M_{\alpha\beta}\psi_{\beta,a}(u) - (1/g_0^2)\sum_p \chi(g_p),$$

where the first sum runs over $u\in\mathbb{Z}_0^3$, $\epsilon=\pm 1$, and $\mu=0,1,2$. For $F(\bar\psi,\psi,g)$, the normalized expectations are denoted by $\langle F\rangle$.

We use the same notation and convention as appears in Ref. [5], and adapt the treatment of symmetries given in Refs. [6,7]. Here, we recall that the Fermi (one-flavor quark and antiquark) fields $\psi_{\alpha,a}(u)$, where $a=1,2,3$ is the color index, $\alpha=1,2\equiv +,-$ is the *spin* index, and $u=(u^0,\vec u)=(u^0,u^1,u^2)$, are defined on the lattice with half-integer

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time coordinates $u\in\mathbb{Z}_0^3\equiv\mathbb{Z}_{1/2}\times\mathbb{Z}^2$, where $\mathbb{Z}_{1/2}=\{\pm 1/2, \pm 3/2, \dots\}$. Letting e^{μ}, $\mu=0,1,2$, denote the unit lattice vectors, there is a gauge group matrix $U(g_{u+e^{\mu},u})=U(g_{u,u+e^{\mu}})^{-1}$ associated with the directed bond $u,u+e^{\mu}$, and we drop U from the notation. For the construction of the physical Hilbert space $\mathcal{H}$ including the e-m operators we refer the reader to Ref. [5].

To search for a meson-baryon bound state, we briefly recall some spectral results for the meson and baryon (antibaryon) particles. The meson fields (see Ref. [7]) are given by $\Pi(u)=(1/\sqrt{3})\bar{\psi}_{-,a}(u)\psi_{+,a}(u)$ and $\mu(u)=(1/\sqrt{3})\psi_{-,a}(u)\bar{\psi}_{+,a}(u)$; the baryon fields are given by $\phi_{-}(u)=\frac{1}{6}\epsilon_{abc}\psi_{-,a}(u)\psi_{-,b}(u)\psi_{-,c}(u)=\psi_{-,1}(u)\psi_{-,2}(u)\psi_{-,3}(u)$ and $\bar{\phi}_{-}(u)=\frac{1}{6}\epsilon_{abc}\bar{\psi}_{-,a}(u)\bar{\psi}_{-,b}(u)\bar{\psi}_{-,c}(u)=\bar{\psi}_{-,1}(u)\bar{\psi}_{-,2}(u)\bar{\psi}_{-,3}(u)$. By charge conjugation symmetry (see Ref. [5]) we have $C\bar{\phi}_{-}(\phi_{-})=i\phi_{+}(-i\bar{\phi}_{+})$ and $C\Pi(\mu)=-\Pi(-\mu)$ where ϕ_{+} ($\bar{\phi}_{+}$) is obtained from ϕ_{-} ($\bar{\phi}_{-}$), replacing the $-$ spin index by $+$ in the single fermion fields. We refer to $\bar{\phi}_{+}(u)$ as the baryon antiparticle and we recall from Ref. [9] that mesons are their own antiparticles. From now on, we suppress the index $-$ from $\bar{\phi}_{-}$. We note that Π, $\bar{\phi}$ generate particles and μ,ϕ are auxiliary fields that enter in the definition of the two-point functions. Considering the Feynman-Kac formula (see Ref. [5]) for $(\Phi^j(1/2,\vec{x}_1), T_0^{|x^0|-1}\Phi^j(1/2,\vec{x}_2))_{\mathcal{H}}$, $x^0\neq 0$, where $j=m$ ($j=b$) refers to mesons (baryons) and we have $\Phi^m=\Pi$ ($\Phi^b=\bar{\phi}$), we are led to define the associated two-point correlation function (χ is the characteristic function, the asterisk denotes complex conjugation) for mesons, $G^m(u^0,\vec{x}_1;v^0,\vec{x}_2)=\chi_{u^0\leqslant v^0}\langle\mu(u^0,\vec{x}_1)\,\Pi(v^0,\vec{x}_2)\rangle+\chi_{u^0>v^0}\langle\Pi(u^0,\vec{x}_1)\mu(v^0,\vec{x}_2)\rangle^*$, and that for baryons, $G^b(u^0,\vec{x}_1;v^0,\vec{x}_2)=-\chi_{u^0\leqslant v^0}\langle\phi(u^0,\vec{x}_1)\bar{\phi}(v^0,\vec{x}_2)\rangle+\chi_{u^0>v^0}\langle\bar{\phi}(u^0,\vec{x}_1)\phi(v^0,\vec{x}_2)\rangle^*$, where $x^0=v^0-u^0\in\mathbb{Z}$. The antibaryon two-point function is obtained from G^b by the application of charge conjugation symmetry as in Ref. [5]. By translation invariance and with manipulation of notation, $G^j(u,v)=G^j(u-v)$. $G^j(x)$, $x^0\neq 0$, admits the spectral representation $G^j(x)=\int_{-1}^{1}\int_{\mathbf{T}^2}(\lambda^0)^{|x^0|-1}e^{i\vec{\lambda}\cdot\vec{x}}d_{\lambda^0}\alpha^j_{\vec{\lambda}}(\lambda^0)d\vec{\lambda}$, where $d_{\lambda^0}\alpha^j_{\vec{\lambda}}(\lambda^0)d\vec{\lambda}=d_{\lambda^0}d_{\vec{\lambda}}(\Phi^j,\mathcal{E}(\lambda^0,\vec{\lambda})\Phi^j)_{\mathcal{H}}$ with $\Phi^j\equiv\Phi^j(1/2,\vec{0})$ and $\mathcal{E}$ is the product of the spectral families for the energy and momentum component operators. For its Fourier transform $\tilde{G}^j(p)=\Sigma_{x\in\mathbb{Z}^3}e^{-ip\cdot x}G^j(x)$, $p=(p^0,\vec{p})\in\mathbf{T}^3$, we get $\tilde{G}^j(p)=\tilde{G}^j(\vec{p})+(2\pi)^2\int_{-1}^{1}\int_{\mathbf{T}^2}f(p^0,\lambda^0)\,\delta(\vec{p}-\vec{\lambda})d_{\lambda^0}\alpha^j_{\vec{\lambda}}(\lambda^0)d\vec{\lambda}$, where $f(x,y)\equiv(e^{ix}-y)^{-1}+(e^{-ix}-y)^{-1}$, $\tilde{G}^j(\vec{p})=\Sigma_{x\in\mathbb{Z}^2}e^{-i\vec{p}\cdot\vec{x}}G^j(x^0=0,\vec{x})$. The spectral measures admit the following decomposition: $d_{\lambda^0}\alpha^j_{\vec{\lambda}}(\lambda^0)=Z^j(\vec{\lambda})\,\delta(\lambda^0-e^{-w^j(\vec{\lambda})})d\lambda^0+d\nu^j(\lambda^0,\vec{\lambda})$. For the complete characterization of the spectral measure including expressions for dispersions relations $w^j(\vec{p})$, we refer the reader to Ref. [9] for mesons and Ref. [8] for baryons. We just recall that points in the spectrum occur as complex p^0 singularities of $\tilde{G}^j(p)$, for fixed $\vec{p}$, and the meson (baryon) isolated dispersion curves occur as singularities for $p^0=\pm iw(\vec{p})$.

To analyze the existence of meson-baryon $\Pi\bar{\phi}$ bound states, we consider the subspace of states generated by $P_{+}[\Pi(1/2,\vec{x}_1)\bar{\phi}(1/2,\vec{x}_2)]$, where $P_{+}\equiv(1/2)(1+P)$ is the projection onto the parity $(+1)$ subspace, where we recall from Ref. [5] that P is the parity symmetry defined by the action on single fermion fields as $P\psi_{\alpha,a}=(\gamma_0)_{\alpha\beta}\psi_{\beta,a}(\mathcal{P}u)$, $P\bar{\psi}_{\alpha,a}=\bar{\psi}_{\beta,a}(\mathcal{P}u)(\gamma_0)_{\beta\alpha}$, $P(AB+cC)=P(A)P(B)+cP(C)$ ($c\in\mathbb{C}$) where A, B, and C are Grassmann monomials and $\mathcal{P}(u^0,\vec{u})=(u^0,-\vec{u})$. We point out that parity symmetry as defined here is equivalent to a spatial reflection, i.e., a rotation in the space coordinates by the angle π. Later on, we will see that the restriction to this parity $(+1)$ subspace will give us an appropriate decay of the four-point function. From the Feynman-Kac formula, for $x^0\neq 0$, we have $(P_{+}[\Pi(1/2,\vec{u}_1)\bar{\phi}(1/2,\vec{u}_2)],(T^0)^{|x^0|-1}\check{T}^{\vec{x}}P_{+}[\Pi(1/2,\vec{u}_3)\times\bar{\phi}(1/2,\vec{u}_4)])_{\mathcal{H}}=\mathcal{G}(x)$, where $\mathcal{G}(x)=\mathcal{G}(u_1,u_2,u_3+\vec{x},u_4+\vec{x})$, with $x=(x^0=v^0-u^0,\vec{x})\in\mathbb{Z}^3$ and, for $u_1^0=u_2^0=u^0$ and $u_3^0=u_4^0=v^0$,

$$\begin{aligned}\mathcal{G}(u_1,u_2,u_3,u_4)&\\=&-\langle P_{+}[\mu(u_1)\phi(u_2)]P_{+}[\Pi(u_3)\bar{\phi}(u_4)]\rangle\chi_{u^0\leqslant v^0}\\&+\langle P_{+}[\Pi(u_1)\bar{\phi}(u_2)]P_{+}[\mu(u_3)\phi(u_4)]\rangle^*\chi_{u^0>v^0}.\end{aligned}$$

The meson-antibaryon four-point function is obtained from $\mathcal{G}(u_1,u_2,u_3,u_4)$ by the use of charge conjugation symmetry of Ref. [5] exactly as we have done for G^b. Using parity symmetry and observing that the meson and baryon fields, as treated here, obey $P\Pi(u)=-\Pi(\mathcal{P}u)$, $P\bar{\phi}(u)=-\bar{\phi}(\mathcal{P}u)$ and the same for the other composite fields μ and ϕ, we get

$$\begin{aligned}\mathcal{G}(u_1,u_2,u_3,u_4)&\\=&-\langle P_{+}[\mu(u_1)\phi(u_2)]\Pi(u_3)\bar{\phi}(u_4)\rangle\chi_{u^0\leqslant v^0}\\&+\langle P_{+}[\Pi(u_1)\bar{\phi}(u_2)]\mu(u_3)\phi(u_4)\rangle^*\chi_{u^0>v^0}.\end{aligned}$$

We now give a rough description of our method before going into detail. We first obtain a spectral representation for $\mathcal{G}(x)$, and its Fourier transform $\tilde{\mathcal{G}}(k)$. In this way, we can relate complex k singularities in $\tilde{\mathcal{G}}(k)$ to the e-m spectrum. Next, using a lattice BS equation in the ladder approximation (see below), we look for the singularities of $\tilde{\mathcal{G}}(p)$ below the meson-baryon threshold ($\sim -5\ln\kappa$).

Taking the Fourier transform and inserting the spectral representations for T_0, T_1, and T_2, we have

$$\begin{aligned}\tilde{\mathcal{G}}(k)=&\tilde{\mathcal{G}}(\vec{k})+(2\pi)^2\int_{-1}^{1}\int_{\mathbf{T}^2}f(k^0,\lambda^0)\,\delta(\vec{k}-\vec{\lambda})\\&\times d_{\lambda^0}d_{\vec{\lambda}}(P_{+}[\Pi(1/2,\vec{u}_1)\bar{\phi}(1/2,\vec{u}_2)],\mathcal{E}(\lambda^0,\vec{\lambda})\\&\times P_{+}[\Pi(1/2,\vec{u}_3)\bar{\phi}(1/2,\vec{u}_4)])_{\mathcal{H}},\end{aligned}$$

where $\tilde{\mathcal{G}}(\vec{k})=\Sigma_{\vec{x}\in\mathbf{T}^2}e^{-i\vec{k}\cdot\vec{x}}\mathcal{G}(x^0=0,\vec{x})$. The singularities in $\tilde{\mathcal{G}}(k)$, for $k=(k^0=i\chi,\vec{k}=0)$ and $e^{\pm\chi}\leqslant 1$, are points in the mass spectrum, i.e., the e-m spectrum at system momentum zero.

To analyze $\tilde{\mathcal{G}}(k)$, we follow the method of analysis for spin models to make the notation closer to that of Ref. [10]. We relabel the time direction coordinates in $\mathcal{G}(x)$ by integer labels, with $u_i^0-1/2=x_i^0$, $\vec{u}_i=\vec{x}_i$, $i=1,\ldots,4$, and write $D(x_1,x_2,x_3+\vec{x},x_4+\vec{x})$, $x_1^0=x_2^0$, $x_3^0=x_4^0$, $x^0=x_3^0-x_2^0$, where x_i and x are points on the $\mathbb{Z}^3$ lattice. Now we pass to difference coordinates and then to lattice relative coordinates $\xi=x_2-x_1$, $\eta=x_4-x_3$, and $\tau=x_3-x_2$ to obtain $D(x_1,x_2,x_3+\vec{x},x_4+\vec{x})=D(0,x_2-x_1,x_3-x_1+\vec{x},x_4-x_1+\vec{x})\equiv D(\vec{\xi},\vec{\eta},\tau+\vec{x})$ and $\tilde{\mathcal{G}}(k)=e^{i\vec{k}\cdot\vec{\tau}}\hat{D}(\vec{\xi},\vec{\eta},k)$, where $\hat{D}(\vec{\xi},\vec{\eta},k)=\Sigma_{\tau\in\mathbb{Z}^3}D(\vec{\xi},\vec{\eta},\tau)e^{-ik\cdot\tau}$. Explicitly, we have

$$
\begin{aligned}
D(x_1,x_2,x_3,x_4)=&-\langle P_+[\mu(x_1^0+1/2,\vec{x}_1)\phi(x_2^0+1/2,\vec{x}_2)]\\
&\times\Pi(x_3^0+1/2,\vec{x}_3)\overline{\phi}(x_4^0+1/2,\vec{x}_4)\rangle\chi_{x_2^0\leqslant x_3^0}\\
&+P_+[\Pi(x_1^0+1/2,\vec{x}_1)\overline{\phi}(x_2^0+1/2,\vec{x}_2)]\\
&\times\mu(x_3^0+1/2,\vec{x}_3)\phi(x_4^0+1/2,\vec{x}_4)\rangle^*\chi_{x_2^0>x_3^0}.
\end{aligned}
$$

The point of all this is that the singularities of $\tilde{\mathcal{G}}(k)$ are the same as those of $\hat{D}(\vec{\xi},\vec{\eta},k)$ and the BS equation for $\hat{D}(\vec{\xi},\vec{\eta},k)$ and its analysis are familiar and have been treated before in Ref. [10].

The BS equation in terms of kernels is

$$
\begin{aligned}
D(x_1,x_2,x_3,x_4)=&D_0(x_1,x_2,x_3,x_4)+\int D_0(x_1,x_2,y_1,y_2)\\
&\times K(y_1,y_2,y_3,y_4)D(y_3,y_4,x_3,x_4)\\
&\times\delta(y_1^0-y_2^0)\delta(y_3^0-y_4^0)dy_1dy_2dy_3dy_4;\\
&x_1^0=x_2^0,x_3^0=x_4^0,
\end{aligned}
$$

where $D_0(x_1,x_2,x_3,x_4)=(1/2)[G^m(x_1-x_3)G^b(x_2-x_4)+G^m(\mathcal{P}x_1-x_3)G^b(\mathcal{P}x_2-x_4)]$, and we use a continuum notation for sums over lattice points. D, D_0, and $K=D_0^{-1}-D^{-1}$ are to be taken as matrix operators acting on $\ell_2^r(\mathcal{A})$, the subspace of $\ell_2(\mathcal{A})$ generated by vectors that are invariant by the spatial reflection $(x_1,x_2)\rightarrow(\mathcal{P}x_1,\mathcal{P}x_2)$, where $\mathcal{A}=\{(x_1,x_2)\in\mathbb{Z}^3\times\mathbb{Z}^3/x_1^0=x_2^0,\ \vec{x}_1\neq\vec{x}_2\}$. In other words, a vector that belongs to $\ell_2^r(\mathcal{A})$ has necessarily the form $c[f(x_1,x_2)+f(\mathcal{P}x_1,\mathcal{P}x_2)]$ $(c\in\mathbb{C})$ with $f(x_1,x_2)\in\ell_2(\mathcal{A})$. In terms of the $(\vec{\xi},\vec{\eta},\tau)$ relative coordinates and taking the Fourier transform in τ only, the BS equation becomes (see Ref. [10]) $\hat{D}(\vec{\xi},\vec{\eta},k)=\hat{D}_0(\vec{\xi},\vec{\eta},k)+\int\hat{D}_0(\vec{\xi},\vec{\xi}',k)\hat{K}(-\vec{\xi}',-\vec{\eta}',k)\hat{D}(\vec{\eta}',\vec{\eta},k)d\vec{\xi}'d\vec{\eta}'$. With k fixed, $\hat{D}(\vec{\xi},\vec{\eta},k)$, etc., is taken as a matrix operator on $\ell_2(\mathcal{B})$, where $\mathcal{B}=\mathbb{Z}^2\setminus\{0\}$; for $k=(k^0,\vec{k}=\vec{0})$ on the even subspace of $\ell_2(\mathcal{B})$. $\hat{K}(-\vec{\xi}',-\vec{\eta}',k)$, in general, acts as an energy-dependent nonlocal potential in the nonrelativistic lattice Schrödinger operator analogy.

We now explain the reason behind the chosen state $P_+[\Pi(1/2,\vec{x}_1)\overline{\phi}(1/2,\vec{x}_2)]$. We want a temporal decay faster than the baryon-meson decay, which is $\kappa^{5|x_3^0-x_1^0|}$, in order to avoid complications due to an energy-dependent potential. By the hyperplane decoupling method (see Ref. [10]) one can show, using the fact that the state above is an eigenvector of parity symmetry with eigenvalue $(+1)$, that K has the decay $\kappa^{7|x_3^0-x_1^0|}$. To obtain the decay $\kappa^{7|x_3^0-x_1^0|}$ we need first to show that $\partial_0^nK=0$ $(n=0,1,2,3)$. The cases $n=0,1,2$ are easy since we can use a gauge integral over a single field and imbalance of fermion fields that appears in the expectations, to show, respectively, $\partial_0^1K=0$ and $\partial_0^nK=0$ $(n=0,2)$. To prove that $\partial_0^3K=0$ we need to show first $\partial_0^3D=0$, which at first sight is not clear. We give below a brief calculation in order to show that $\partial_0^3D=0$. The hyperplane decoupling method consists of replacing the κ in the action S by the complex parameter κ_p for all bonds connecting the hyperplanes $u^0=p$ and $u^0=p+1$. For $u^0\leqslant p<v^0$ with $x_1^0=x_2^0=u^0\leqslant x_3^0=x_4^0=v^0$ we have $\partial_0^3D(x_1,x_2,x_3,x_4)=6\Sigma_{\vec{w}}\langle\mu(x_1)\phi(x_2)\overline{\phi}(p,\vec{w})\rangle^{(0)}\langle\phi(p+l,\vec{w})P_+[\Pi(x_3)\overline{\phi}(x_4)]\rangle^{(0)}$ where $\langle\cdots\rangle^{(0)}$ is obtained from $\langle\cdots\rangle$ by setting $\kappa_p=0$ and ∂_0 means the κ_p derivative evaluated at $\kappa_p=0$. Consider the second term in the expression above summed over translations of $\vec{x}_3$ and $\vec{x}_4$ to put the system space momentum at zero with $\vec{w}$ fixed, i.e., $F(x_3,x_4)=\int\langle\phi(p+l,\vec{w})P_+[\Pi(x_3+\vec{z})\overline{\phi}(x_4+\vec{z})]\rangle^{(0)}d\vec{z}$. Using parity symmetry, followed by the change of variables $\vec{z}'=\vec{z}+2\vec{w}$, we get $-F(x_3,x_4)$ from which we conclude that $F(x_3,x_4)=0$ and hence $\partial_0^3D=0$. Without this modification we would have $\partial_0^3K\neq 0$ and using Cauchy bounds for the derivatives of K we would get only the temporal decay $\kappa^{3|x_3^0-x_1^0|}$. This improved decay together with the control of perturbations to the ladder approximation lead to a rigorous solution of the BS equation and two-particle spectral results for the complete model [10].

We now obtain what we call a ladder approximation L to K (see Ref. [8]). For baryon-baryon (meson-meson) [8] ([9]) states, the leading order contribution to K, i.e., the ladder approximation, was found to be of the order κ^2 and local (nonlocal), where we recall that the local potential is given by contributions to $L(x_1,x_2,x_3,x_4)$ with $x_1=x_3$ and $x_2=x_4$. In both cases the points $x_1=x_3=0$, $x_2=x_4=x$ $(|x|=1)$ for baryon-baryon states and $x_i=0$, $x_j=x_k=x_\ell=x$ $(|x|=1)$ $(i,j,k,\ell\in\{1,2,3,4\}$ and all distinct) for meson-meson states are connected by two overlapping bonds with opposite orientation. In our case, L is given by the κ^3 contribution to $D_0^{-1}D^TD_0^{-1}$, the first nonvanishing term in the Neumann series for $K=D_0^{-1}-[D_0+D^T]^{-1}$, where $D^T=D-D_0$ is the truncated four-point function. The κ^2 contribution to K comes if we take the same site for ϕ (Π) and $\overline{\phi}(\mu)$. Note that, this is the only allowed contribution at this order. If we consider, for example, $x_1=x_4=0$, $x_2=x_3=x$ $(|x|=1)$ we would have zero due to imbalance of fermion-fields appearing in the expectations. We get the contribution $\frac{1}{3}(-\kappa/2)^2 \langle \psi_{-,a}\overline{\psi}_{+,a}\overline{\psi}_{-,b}\psi_{+,b}\overline{\psi}_{\alpha_1,a_1}\psi_{\beta_2,a_1}(0)\rangle^0 \times \langle \psi_{\beta_1,a_2}\overline{\psi}_{\alpha_2,a_2}\phi\overline{\phi}(e^j)\rangle^0 \Gamma^{\epsilon e^j}_{\alpha_1\beta_1}\Gamma^{-\epsilon e^j}_{\alpha_2\beta_2}$, where $\langle\cdots\rangle^0$ is the expectation $\langle\cdots\rangle$ setting $\kappa=0$. We now analyze the first factor of the last expression. For $a=b$, by saturation in the color we get $a_1\neq a$ and hence $\alpha_1=\beta_2$. Using the following property of Γ matrices, $\Gamma^{\epsilon e^j}\Gamma^{-\epsilon e^j}=0$, we get zero for the first factor. For $a\neq b$ the first factor is zero by fermion component imbalance for $a_1=1,2,3$. We therefore get zero for the κ^2 contribution to K, hence, excluding a meson exchange interpretation between two pairs of particles separated by one lattice unit. Proceeding as in the calculation above we get the κ^3 contribution, which is local and given by points $x_1=x_4=0$, $x_2=x_3=x$ $(|x|=1)$ connected by three overlapping bonds with the same orientation. The $\mathcal{O}(\kappa^3)$ contribution to D comes from computations of three-fermion (and three-antifermion) exchange contributions between two pairs of meson and baryon particles such that we cannot identify them with a baryon [recall that a baryon field is given by $\phi_-(u)=\psi_{-,1}(u)\psi_{-,2}(u)\psi_{-,3}(u)$] due to the spin index [we get, e.g., $\psi_{+,1}(u)\psi_{-,2}(u)\psi_{-,3}(u)$]. We obtain, for $c'(\kappa)=-\frac{3}{16}\kappa^3$, $L(x_1,x_2,x_3,x_4)=c'(\kappa)\Sigma_{j=1,2;\epsilon=\pm1}\delta(x_2-x_1-\epsilon e^j)[\delta(x_4-x_1)\delta(x_3-x_2)+\delta(x_4+x_1)\delta(x_3+x_2)]$ and, as $\hat{L}$ acts on $\ell_2^r(\mathcal{A})$, we have $L(x_1,x_2,x_3,x_4)=2c'(\kappa)\Sigma_{j=1,2;\epsilon=\pm1}\delta(x_2-x_1-\epsilon e^j)\delta(x_4-x_1)\delta(x_3-x_2)$, and hence, in relative coordinates, $\hat{L}(\vec{\xi},\vec{\eta},k^0)=2c'(\kappa)\Sigma_{j,\epsilon}\delta(\vec{\xi}-\vec{\eta})\delta(\vec{\xi}-\epsilon e^j)$, where we use the abbreviated notation k^0 for $k=(k^0,\vec{k}=\vec{0})$, which we will omit below. In the lattice Schrödinger operator analogy, $\hat{L}$ corresponds to a local energy-independent and repulsive potential.

The BS equation in the ladder approximation is $\hat{D}(\vec{\xi},\vec{\eta})=\hat{D}_0(\vec{\xi},\vec{\eta})+2c'(\kappa)\Sigma_{j,\epsilon}\hat{D}_0(\vec{\xi},\epsilon e^j)\hat{D}(\epsilon e^j,\vec{\eta})$ and we now obtain its solution. Following Ref. [8] we solve the equation above for the variables $\hat{D}(e^j,\vec{\eta})$ $(j=1,2)$ to get [with $c(\kappa)=4c'(\kappa)$] $\hat{D}(\vec{\xi},\vec{\eta})=\hat{D}_0(\vec{\xi},\vec{\eta})+c(\kappa)\Sigma_{jk}\hat{D}_0(\vec{\xi},e^j)(\mathcal{M}_{jk}/W)\hat{D}_0(e^k,\vec{\eta})$ where $\mathcal{M}$ denotes the 2×2 matrix with entries $\mathcal{M}_{11}=\mathcal{M}_{22}=1-c(\kappa)\hat{D}_0(e^1,e^1)$, $\mathcal{M}_{12}=\mathcal{M}_{21}=-c(\kappa)\hat{D}_0(e^1,e^2)$. We see that the only singularities of $\hat{D}(\vec{\xi},\vec{\eta})$ on the imaginary k^0 axis, and below the meson-baryon threshold, occur as zeros of $W(k^0)\equiv\det\mathcal{M}$.

We now obtain a representation for $\hat{D}_0$ that is to be used to determine a bound state. We maintain only the product of one meson and one baryon contributions to $\hat{D}_0(\vec{\xi},\vec{\eta},k^0)$ to get $\hat{D}_0(\vec{\xi},\vec{\eta},k^0)\simeq\int_{\mathbf{T}^2}M(\vec{p},k^0)\cos\vec{p}\cdot\vec{\xi}\cos\vec{p}\cdot\vec{\eta}d\vec{p}$ where

$$M(\vec{p},k^0)=(2\pi)^{-2}\tilde{G}^m(\vec{p})\tilde{G}^b(\vec{p})+(2\pi)^2\frac{Z^m(\vec{p})Z^b(\vec{p})}{e^{ik^0}-e^{-w^m(\vec{p})-w^b(\vec{p})}}.$$

To determine the bound state we must search for the zeros of $W(k^0)=1-c(\kappa)[\hat{D}_0(e^1,e^1)\mp\hat{D}_0(e^1,e^2)]$. Following the same analysis of Ref. [8], one can use the Cauchy-Schwarz inequality to show $|\hat{D}_0(e^1,e^2,k^0=i\chi)|\leqslant\hat{D}_0(e^1,e^1,k^0=i\chi)$ and hence exclude the bound state from the spectrum, since, in our case this gives $W(k^0)\geqslant1$.

We now give the representation of $\hat{D}_0$ that is to be used to establish the connection between the BS equation and the lattice Schrödinger resolvent equation. We find $Z^j(\vec{p})\simeq(2\pi)^{-2}e^{-w^j(\vec{p})}$ $(j=m,b)$; $w^m(\vec{p})\simeq m^m_\kappa+c_m\kappa^2[-\tilde{\Delta}(\vec{p})]$, $c_m=1/4$ (see Ref. [9]) and $w^b(\vec{p})\simeq m^b_\kappa+c_b\kappa^3[-\tilde{\Delta}(\vec{p})]$, $c_b=1/8$ (see Ref. [5]) where $-\tilde{\Delta}(\vec{p})=2\Sigma_{j=1,2}(1-\cos p^j)$ and m^m_κ (m^b_κ) is the meson (baryon) mass. Letting $k^0=i\chi=i(m^m_\kappa+m^b_\kappa-\overline{\epsilon})$ so that $\overline{\epsilon}>0$ is the meson-baryon binding energy, we have, if $\overline{\epsilon}\ll1$,

$$\hat{D}_0(\vec{\xi},\vec{\eta},k^0)\simeq(2\pi)^{-2}\int_{\mathbf{T}^2}\frac{\cos\vec{p}\cdot\vec{\xi}\cos\vec{p}\cdot\vec{\eta}}{(c_m\kappa^2+c_b\kappa^3)[-\tilde{\Delta}(\vec{p})]+\overline{\epsilon}}d\vec{p}.$$

In the context of the approximations above in the ladder BS equation, we make the identifications $a=c_m\kappa^2+c_b\kappa^3$, $\lambda=\frac{3}{8}\kappa^3$, $V=\Sigma_{j,\epsilon}\delta(\vec{\xi}-\vec{\eta})\delta(\vec{\xi}-\epsilon e^j)$ and $z=-\overline{\epsilon}$ in the lattice Schrödinger resolvent equation, $(H-z)^{-1}=(H_0-z)^{-1}-\lambda(H_0-z)^{-1}V(H-z)^{-1}$, where $H=H_0+\lambda V$, $H_0=-a\Delta$ [Δ is the lattice Laplacian on $\ell_2(\mathbb{Z}^2)$], and V is a local potential acting on the even subspace of $\ell_2(\mathbb{Z}^2\setminus\{0\})$. Thus a positive λ corresponds to a repulsive potential.

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